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- Linear quadratic regulator to design the optimal controller

-s/w: matlab, symbolic math

1. Problem:

where

Find the optimal controller to minimize in stead state.

2.Solution:

2.1 Theory

In steady state optimal controller is given by

where is the **Positive solution** of the following Algebraic Riccati Equation,

(in the transient state, the Riccati equation is

But in the steady state, the final state weighting is irrelevant.)

2.2 Numerical Solution:

The equation (2) is a quadratic scalar equation, so you may have a famous formula to get the solution. Yap. It is good. However let’s solve it using matlab, **symbolic math**

clear all; clc

syms p q r a b

f = q + p\*a + a\*p - p\*b\*r^-1\*b\*p;

S =solve(f==0,p)

pretty(S)

Then you may have two solutions, but the P should be positive(why?), the only solution is

2.3 The analysis

Now the solution is obtained. You may substitute to your own alphabets. I will continue what we are doing.

* Close loop pole

1. The original system is unstable

The closed loop system matrix(in this case scalar) is . let’s me particular values of then

u = r^-1\*b\*p;

Ac = a-S(1);

subs(Ac,[q r a b ],[1 1 1 1])

Then the closed loop pole is . Make sense. It is a negative real part to ensure the asymptotic stability. Hence the optimal feedback controller stabilizes the unstable original system. Good. Now if , the close loop pole is still …

1. Related to the system matrix

Let’s assume the weighting matrix is fixed. Now how about different ?

If , then the closed loop system matrix . Strange?...Compare to PID controller…

2.4 The control design point of view

So the optimal controller is stabilize the closed system whether the system is stable or not. This is the most important things in design point of view.

Now we may have another parameters which can be adjusted to your control point of views.

The Weighting matrix . How to choose these matrices? There is no unique way. Depends.

If your are to be a good control engineer, you should be familiar to simulation.